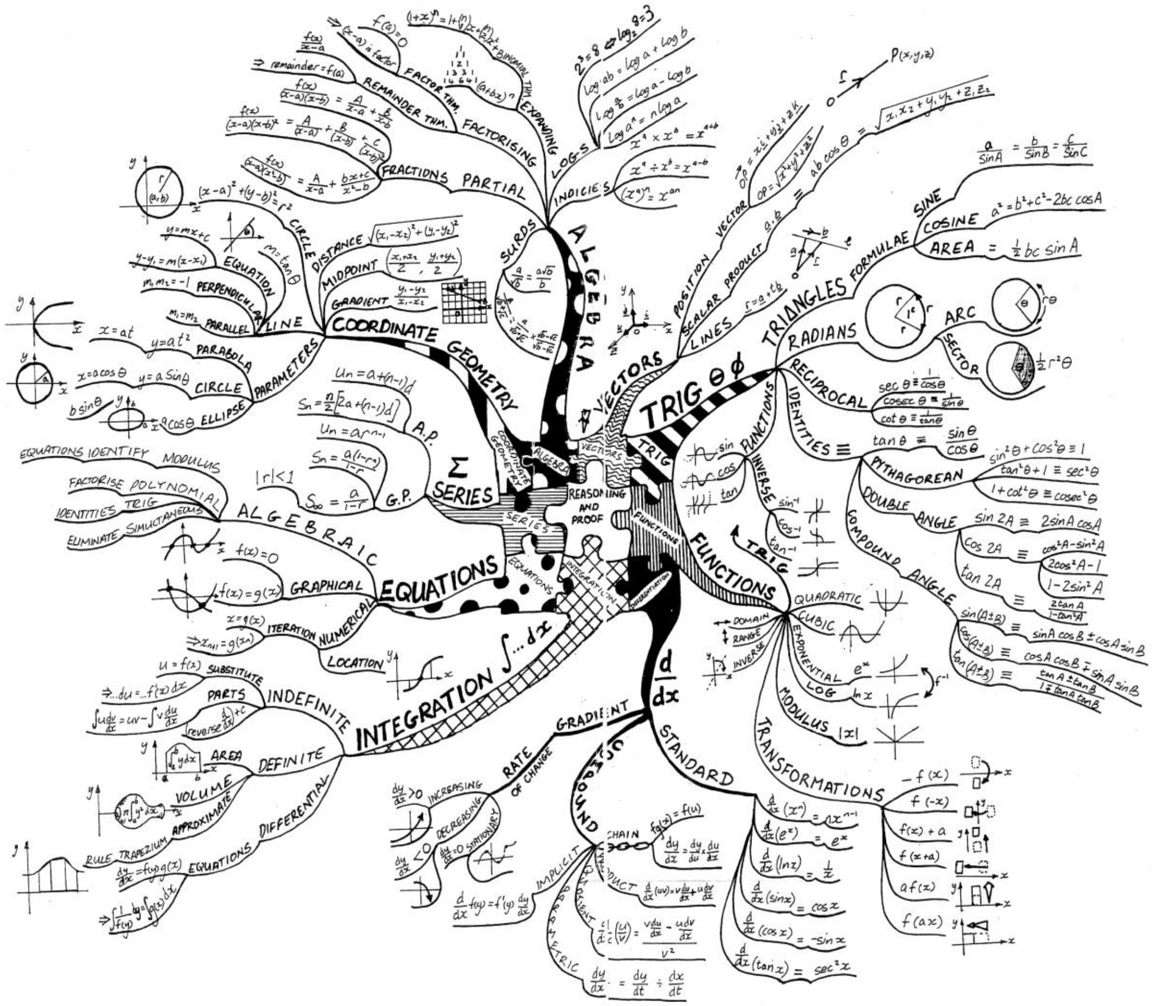
4. Changing the Subject of a Formula



This booklet belongs to ______________________________________

Aims and Objectives

Changing the subject of a formula is the same process as solving equations, in that we are rearranging a formula by using the balance model seen in booklet 2. When you have completed this booklet, you should

- be able to re – write a formula in terms of another letter

1. The Underlying Principle

A formula is a mathematical equation containing two or more letters.

Suppose that you have a formula such as $2x = 3a$. We could write this formula as

$$x = \frac{3a}{2}$$

in which case we would say that x is the **subject of the formula**, or that x is given/written in terms of a.

Alternatively we could have written the formula as

$$a = \frac{2x}{3}$$

in which case we would say that a is the subject of the formula, or that a is given/written in terms of x.

2. Changing the Subject of a Formula

The rules are the same as when we solved equations except we do not end up with a solution but with another formula.

Examples

1. In each case, make x the subject of the formula

a) $a + x = b + c$

b) $a + 3x = b + c$

c) $ax = b + c$

Now try these

2. In each case, make x the subject of the formula

a) $x + a = b$

b) $x + b = a - b$

c) $a - x = b$

d) $-ax = 2b$

e) $ax + c = b + c$

3. In each case, make the letter given at the end, the subject of the formula

a) $y = mx + c, \;\; c$

b) $s = vt, \;\; t$

c) $V = IR, \;\; R$

d) $v^2 = u^2 + 2as, \;\; s$

e) $v = u + at, \;\; a$

3. Formulae with Brackets and fractions

If the letter you are rearranging for is in a bracket, then you can approach the rearranging in one of two ways.

Example

1. Make x the subject of the formula

a) $a(x + b) = c$

either, expand the bracket and rearrange

or, divide and rearrange

To remove fractions in formulae, first multiply by the appropriate number or letters, but remember that the fraction bar acts as a bracket, so put the brackets in when appropriate.

Examples

2. Make x the subject of the formula

a) $\frac{a}{x} = b$

b) $\frac{x}{a} = 1 + \frac{1}{b}$

c) $\frac{x+p}{p} = \frac{p}{q} + \frac{q}{p}$

Now you try these ones

3. Make x the subject of the formula

a) $2(x+a) = b$

b) $\frac{1}{2}x - a = 4b$

c) $\frac{a}{x} = b + c$

d) $\frac{x}{a} = \frac{a+b}{b}$

e) $a(x+c) = c(a+b)$

4. In each case, make the letter given at the end, the subject of the formula

a) $v = \frac{d}{t}, \;\; d$

b) $s = ut + \frac{1}{2}at^2, \;\; u$

c) $\frac{v-u}{a} = t, \;\; u$

d) $s = \frac{1}{2}t(u + v)$, u

e) $y = a^2x + b^2$, x

4. The Need for Factorising

The aim of rearranging is to manipulate the formula so that all the terms involving the letter you want is on one side of the equation, and everything else is on the other. That is straight forward if there is only one term involving the letter you are rearranging for. If however there is more than one term, then you need to add an extra step, which is to factorise.

Example

1. Make x the subject of the formula

a) $ax + bx = ab$

b) $ax + b = cx + d$

c) $a(x + c) = b(x + c)$

d) $y = \frac{x + a}{x - a}$

e) $\frac{a}{x} + b = c$

Try these ones

2. Make x the subject of the formula

a) $x + xy = y$

b) $x = y + xy$

c) $px - qx = p$

d) $ax = bx + c$

e) $hx = k - kx$

f) $p(x + q) = 2q(x + p)$

g) $y = 1 - \frac{1}{x}$

h) $y = \frac{x + 1}{x - 1}$

i) $y = 1 - \frac{1}{x - 1}$

3. In each case, make the letter given at the end, the subject of the formula

a) $A = P + \frac{1}{100}PRT, \; T$

b) $s = \frac{1}{2}d(a + l), \; a$

c) $s = \frac{1}{2}n(2a + (n - 1)d), \qquad d$

d) $E = \frac{1}{2}mv^2 + mgh, \; m$

5. Formulae Involving Roots and Powers

You have seen that x^2 means $x \times x$. If we want to solve the equation

$$x^2 = 16$$

we would need to find a number that when multiplied by itself gives us the answer 16. The operation that allows us to do this is called the **square root**. In this case we find the square root of 16, written $\sqrt{16}$. You may know the answer already as 4, but notice that this answer is positive, that is $\sqrt{16} = +4$. However it is possible that -4 can also be the answer to the equation

$$x^2 = 16$$

as $(-4) \times (-4) = 16$

in which case we would write$-\sqrt{16} = -4$.

So in conclusion the solutions to the equation

$$x^2 = 16$$

are

$$x = \pm\sqrt{16} = \pm 4$$

and in general the solution to the equation

$$x^2 = a$$

is

$$x = \pm\sqrt{a}$$

Cube roots are less complicated than square roots as the solution to the equation

$$x^3 = a$$

is the cube root of a, written $x = \sqrt[3]{a}$, and there is only one solution to this.

So in conclusion, when rearranging an equation or a formula you will often need to 'undo' a square or a cube. In order to do this you will need to square root or cube root. But remember that when you square root you need to put a $\pm$ in front of the root sign.

To 'undo' square roots or cube roots, you need to square or cube.

Examples

1. Make x the subject of the formula

a) $\sqrt{x} - 1 = a$

b) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

c) $\sqrt{x+1} = a$

Now try these ones

2. Make x the subject of the formula

a) $\sqrt{x} + 1 = a$

b) $x^2 - y^2 = a^2$

c) $\sqrt[3]{x} - a = 1$

3. In each case, make the letter given at the end, the subject of the formula

a) $A = 4\pi r^2, \; r$

b) $V = \frac{4}{3}\pi r^3, \; r$

c) $ay^2 = x^3, \; y$

d) $V = \pi r^2 h, \; r$

Further Practice

1. In each case, make the letter x, the subject of the formula

a) $ax + a = b + c$

b) $a + 2x = b$

c) $ax - b = 2b$

d) $ax - b + c = b$

e) $a(x + b) = b$

f) $\frac{a(x + b)}{c} = d$

g) $\frac{x}{a} = \frac{a}{b}$

h) $x + y = xy$

i) $rs + sx - tx = u$

j) $ax + b(x - a) = 0$

k) $y = \frac{x+1}{x}$

l) $\frac{x}{a} = \frac{x}{b} - 1$

m) $y = \frac{x+k}{x+2k}$

n) $x^2 - 1 = a^2$

o) $\sqrt{x^2 + a^2} = a$

p) $x^3 - a^3 = 1$

2. In each case, make the letter given at the end, the subject of the formula

a) $y = mx + c, \;\; m$

b) $v = u + at, \;\; u$

c) $y = a^2x + b^2, \;\; x$

d) $v = \frac{d}{t}, \;\; t$

e) $s = ut + \frac{1}{2}at^2,\ \ a$

f) $\frac{v-u}{a} = t,\ \ v$

g) $\frac{A-a^2}{a} = 4b,\ \ A$

h) $A = P + \frac{1}{100}PRT,\ \ P$

i) $s = \frac{1}{2}d(a+l),\ \ d$

j) $s = \frac{1}{2}(u+v)t,\ \ t$

k) $\frac{x^3}{a^3} - \frac{y^2}{b^2} = 1,\ \ x$

l) $E = \frac{1}{2}mv^2 - \frac{1}{2}mu^2,\ \ u$

Answers:

1a) $x = \frac{b+c-a}{a}$, b) $x = \frac{b-a}{2}$, $c)\ x = \frac{3b}{a}$,
$d)\ \ x = \frac{2b-c}{a},\ e)\ x = \frac{b}{a} - b$

$f)\ \ x = \frac{cd}{a} - b\ \ g)\ \ x = \frac{a^2}{b}\ \ \ h)\ \ x = \frac{y}{y-1}$,
$i)\ \ x = \frac{u-rs}{s-t}$, j) $x = \frac{ab}{a+b}$

$k)\ x = \frac{1}{y-1},\ l)\ x = \frac{ab}{a-b},\ \ m)\ x = \frac{k-2ky}{y-1}$,
$n)\ \ x = \pm\sqrt{a^2+1},\ o)\ x = 0,\ \ p)\ \ x = \sqrt[3]{1+a^3}$

$2a)\ \ m = \frac{y-c}{x},\ b)\ \ u = v - at,\ c)\ \ x = \frac{y-b^2}{a^2}$,
$d)\ \ t = \frac{d}{v},\ e)\ \ a = \frac{2(s-ut)}{t^2}\ \ f)\ \ v = u + at$,
g) $A = 4ab + a^2,\ h)\ \ P = \frac{100A}{100+RT}\ \ i)\ \ d = \frac{2S}{a+l}$,
$j)\ \ t = \frac{2s}{u+v}$, k) $x = \sqrt[3]{a^3 + \frac{a^3y^2}{b^2}}$

$l)\ u = \pm\sqrt{\frac{mv^2 - 2E}{m}}$

Extra

1. Rearrange the following formulae, making x the subject

a) $ax + bx = x(a - b) + c$

b) $s = \frac{1}{2}n(x + y)$

c) $b(x + a) = a$

d) $x(b + c) - a = b$

e) $\frac{x}{a} + \frac{y}{b} = \frac{xy}{ab}$

f) $\frac{x^3}{a^3} + \frac{y^3}{b^3} = 1$

2. In each case, make the letter given at the end, the subject of the formula

a) $s = \dfrac{a}{1-r}$, r

b) $A = 2\pi r(r+h)$, h

c) $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$, f

d) $h = \dfrac{1}{2}gt^2$, t

e) $v^2 = w^2(a^2 - x^2)$, x

f) $V = \dfrac{1}{3}\sqrt{\dfrac{s^3}{8\pi}}$, s

Notes