| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1 (a)** | Systematic (sample) | B1cao | 1.2 |
| **(b)** | In LDS some days have gaps because the data was not recorded | B1 | 2.4 |
| | | | |
| | | | **(2 marks)** |

| Part | Notes |
|---|---|
| **(b)** | B1 a correct explanation |
| | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2** | $\frac{1}{2}r^2(4.8)$ | M1 | 1.1a |
| | $\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e. | A1 | 1.1b |
| | length of minor arc $= 7.5(2\pi - 4.8)$ | dM1 | 3.1a |
| | $= 15\pi - 36 \quad \{a = 15,\, b = -36\}$ | A1 | 1.1b |
| | | **(4)** | |
| **Alt** | $\frac{1}{2}r^2(4.8)$ | M1 | 1.1a |
| | $\frac{1}{2}r^2(4.8) = 135 \Rightarrow r^2 = \frac{225}{4} \Rightarrow r = 7.5$ o.e. | A1 | 1.1b |
| | length of major arc $= 7.5(4.8)\ \{= 36\}$ | | |
| | length of minor arc $= 2\pi(7.5) - 36$ | dM1 | 3.1a |
| | $= 15\pi - 36 \quad \{a = 15,\, b = -36\}$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(4 marks)** |

**Question 2 Notes:**

| | |
|---|---|
| **M1:** | Applies formula for the area of a sector with $\theta = 4.8$; i.e. $\frac{1}{2}r^2\theta$ with $\theta = 4.8$<br>**Note:** Allow M1 for considering ratios. E.g. $\frac{135}{\pi r^2} = \frac{4.8}{2\pi}$ |
| **A1:** | Uses a correct equation $\left(\text{e.g. } \frac{1}{2}r^2(4.8) = 135\right)$ to obtain a radius of 7.5 |
| **dM1:** | Depends on the previous M mark.<br>A complete process for finding the length of the minor arc *AB*, by either<br>• $(\text{their } r) \times (2\pi - 4.8)$<br>• $2\pi(\text{their } r) - (\text{their } r)(4.8)$ |
| **A1:** | Correct exact answer in its simplest form, e.g. $15\pi - 36$ or $-36 + 15\pi$ |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | Identifies an error for student A: They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$ <br> It should be $\frac{\sin\theta}{\cos\theta}=\tan\theta$ | B1 | 2.3 |
| | | **(1)** | |
| **(b)** | (i) Shows $\cos(-26.6°)\neq 2\sin(-26.6°)$, so cannot be a solution | B1 | 2.4 |
| | (ii) Explains that the incorrect answer was introduced by squaring | B1 | 2.4 |
| | | **(2)** | |
| | | | **(3 marks)** |

**Notes:**

**(a)**

**B1:** Accept a response of the type 'They use $\frac{\cos\theta}{\sin\theta}=\tan\theta$. This is incorrect as $\frac{\sin\theta}{\cos\theta}=\tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta=\frac{1}{2}$ not $\tan\theta=2$'

Accept also statements such as 'it should be $\cot\theta=2$'

**(b)**

**B1:** Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta=2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°)=+ve$ and $2\sin(-26.6°)=-ve$ and stating that they therefore cannot be equal.

**B1:** Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x=5$ squared gives $x^2=25$ which has answers $\pm5$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 4(a) | Uses $\cos^2 x = 1-\sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1-\sin^2 x)$ | M1 | 3.1a |
| | $\Rightarrow 12\sin^2 x + \sin x - 1 = 0$ | A1 | 1.1b |
| | $\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$ | M1 | 1.1b |
| | $\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$ | A1 | 1.1b |
| | Uses arcsin to obtain two correct values | M1 | 1.1b |
| | All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$ | A1 | 1.1b |
| | | **(6)** | |
| (b) | Attempts $2\theta - 30° = -19.47°$ | M1 | 3.1a |
| | $\Rightarrow \theta = 5.26°$ | A1ft | 1.1b |
| | | **(2)** | |
| | | **(8 marks)** | |

**Notes:**

**(a)**

**M1**: Substitutes $\cos^2 x = 1-\sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

**A1**: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

**M1**: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

**A1**: $\sin x = \frac{1}{4}, -\frac{1}{3}$

**M1**: Obtains two correct values for their $\sin x = k$

**A1**: All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

**(b)**

**M1**: For setting $2\theta - 30° =$ their "$-19.47\ldots$"

**A1ft**: $\theta = 5.26°$ but allow a follow through on their "$-19.47\ldots$"

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5 | $2\log_4(2-x)-\log_4(x+5)=1$ | | |
| | Uses the power law $\log_4(2-x)^2-\log_4(x+5)=1$ | M1 | 1.1b |
| | Uses the subtraction law $\log_4\dfrac{(2-x)^2}{(x+5)}=1$ | M1 | 1.1b |
| | $\dfrac{(2-x)^2}{(x+5)}=4\rightarrow$ 3TQ in *x* | dM1 | 3.1a |
| | $x^2-8x-16=0$ | A1 | 1.1b |
| | $(x-4)^2=32\Rightarrow x=$ | M1 | 1.1b |
| | $x=4-4\sqrt{2}$ oe only | A1 | 2.3 |
| | | **(6)** | |
| **(6 marks)** | | | |
| **Notes:** | | | |

**M1:** Uses the power law of logs $2\log_4(2-x)=\log_4(2-x)^2$

**M1:** Uses the subtraction law of logs following the above $\log_4(2-x)^2-\log_4(x+5)=\log_4\dfrac{(2-x)^2}{(x+5)}$

Alternatively uses the addition law following use of $1=\log_4 4$ That is $1+\log_4(x+5)=\log_4 4(x+5)$

**dM1:** This can be awarded for the overall strategy leading to a 3TQ in *x*. It is dependent upon the correct use of both previous M's and for undoing the logs to reach a 3TQ equation in *x*

**A1:** For a correct equation in *x*

**M1:** For the correct method of solving their 3TQ =0

**A1:** $x=4-4\sqrt{2}$ or exact equivalent only. (For example accept $x=4-\sqrt{32}$ )

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6(a) | For a correct equation in $p$ or $q$ $p=10^{4.8}$ or $q=10^{0.05}$ | M1 | 1.1b |
| | For $p=$ awrt 63100 or $q=$ awrt 1.122 | A1 | 1.1b |
| | For correct equations in $p$ and $q$ $p=10^{4.8}$ and $q=10^{0.05}$ | dM1 | 3.1a |
| | For $p=$ awrt 63100 and $q=$ awrt 1.122 | A1 | 1.1b |
| | | **(4)** | |
| (b) | (i) The value of the painting on 1st January 1980 | B1 | 3.4 |
| | (ii) The proportional increase in value each year | B1 | 3.4 |
| | | **(2)** | |
| (c) | Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$ | M1 | 3.4 |
| | $=$ awrt $(£)2000000$ | A1 | 1.1b |
| | | **(2)** | |
| | | | **(8 marks)** |

**Notes**

**(a)**

**M1:** For a correct equation in $p$ or $q$ This is usually $p=10^{4.8}$ or $q=10^{0.05}$ but may be $\log q=0.05$ or $\log p=4.8$

**A1:** For $p=$ awrt 63100 or $q=$ awrt 1.122

**M1:** For linking the two equations and forming correct equations in $p$ and $q$. This is usually $p=10^{4.8}$ and $q=10^{0.05}$ but may be $\log q=0.05$ and $\log p=4.8$

**A1:** For $p=$ awrt 63100 and $q=$ awrt 1.122 Both these values implies M1 M1

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**ALT I(a)**

**M1:** Substitutes $t=0$ and states that $\log p=4.8$

**A1:** $p=$ awrt 63100

**M1:** Uses their found value of $p$ and another value of $t$ to find form an equation in $q$

**A1:** $p=$ awrt 63100 and $q=$ awrt 1.122

......................................................................................................................

**(b)(i)**

**B1:** The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

**B1:** The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of $q$.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

**If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled " $p$ is............ " and "$q$ is ................"**

**(c)**

**M1:** For substituting $t=30$ into $V=pq^t$ using their values for $p$ and $q$ or substituting $t=30$ into $\log_{10}V=0.05t+4.8$ and proceeds to $V$

**A1: For awrt either** £1.99 million or £2.00 million. Condone the omission of the £ sign.

Remember to isw after a correct answer

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **7 (a)** | $y = \dfrac{3x-5}{x+1} \Rightarrow y(x+1) = 3x-5 \Rightarrow xy + y = 3x-5 \Rightarrow y+5 = 3x - xy$ | M1 | 1.1b |
| | $\Rightarrow y+5 = x(3-y) \Rightarrow \dfrac{y+5}{3-y} = x$ | M1 | 2.1 |
| | Hence $f^{-1}(x) = \dfrac{x+5}{3-x}, \quad x \in \mathbb{R},\ x \neq 3$ | A1 | 2.5 |
| | | **(3)** | |
| **(b)** | $fg(2) = f(4-6) = f(-2) = \dfrac{3(-2)-5}{-2+1} ;= 11$ | M1 | 1.1b |
| | | A1 | 1.1b |
| | | **(2)** | |
| **(c)** | $g(x) = x^2 - 3x = (x-1.5)^2 - 2.25$. Hence $g_{min} = -2.25$ | M1 | 2.1 |
| | Either $g_{min} = -2.25$ or $g(x) \geqslant -2.25$ or $g(5) = 25 - 15 = 10$ | B1 | 1.1b |
| | $-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$ | A1 | 1.1b |
| | | **(3)** | |
| **(d)** | E.g.<br>• the function g is many-one<br>• the function g is not one-one<br>• the inverse is one-many<br>• $g(0) = g(3) = 0$ | B1 | 2.4 |
| | | **(1)** | |
| | | | **(9 marks)** |

| Question 7 Notes: | |
|---|---|
| **(a)** | |
| **M1:** | Attempts to find the inverse by cross-multiplying and an attempt to collect all the *x*-terms (or swapped *y*-terms) onto one side |
| **M1:** | A fully correct method to find the inverse |
| **A1:** | A correct $f^{-1}(x) = \frac{x+5}{3-x}, \quad x \in \mathbb{R}, \; x \neq 3$, expressed fully in function notation (including the domain) |
| **(b)** | |
| **M1:** | Attempts to substitute the result of $g(2)$ into f |
| **A1:** | Correctly obtains $fg(2) = 11$ |
| **(c)** | |
| **M1:** | Full method to establish the minimum of g.<br>E.g.<br>• $(x \pm a)^2 + b$ leading to $g_{min} = b$<br>• Finds the value of *x* for which $g'(x) = 0$ and inserts this value of *x* back into $g(x)$ in order to find to $g_{min}$ |
| **B1:** | For either<br>• finding the correct minimum value of g<br>(Can be implied by $g(x) \geqslant -2.25$ or $g(x) > -2.25$)<br>• stating $g(5) = 25 - 15 = 10$ |
| **A1:** | States the correct range for g. E.g. $-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$ |
| **(d)** | |
| **B1:** | See scheme |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 8(a) | Uses $-2(3-x)+5=\frac{1}{2}x+30$ | M1 | 3.1a |
| | Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$ | M1 | 1.1b |
| | $x=\frac{62}{3}$ only | A1 | 1.1b |
| | | **(3)** | |
| (b) | Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k\leqslant 11$ | M1 | 2.2a |
| | $\{k:k\in\mathbb{R}, 5<k\leqslant 11\}$ | A1 | 2.5 |
| | | **(2)** | |
| | | | **(5 marks)** |

**Notes:**

**(a)**

**M1:** Deduces that the solution to $\mathrm{f}(x)=\frac{1}{2}x+30$ can be found by solving

$-2(3-x)+5=\frac{1}{2}x+30$

**M1:** Correct method used to solve their equation. Multiplies out bracket/ collects like terms

**A1:** $x=\frac{62}{3}$ only. Do not allow 20.6

**(b)**

**M1:** Deduces that two distinct roots occurs when $y=k$ intersects $y=\mathrm{f}(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k\leqslant 11$

**A1:** Correct solution only $\{k:k\in\mathbb{R}, 5<k\leqslant 11\}$