U6 – Assessment 1 – Solutions

Q1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(x-4)(3x+1)+B(3x+1)+C(x-4)(x-4)\equiv 18x^2-98x+78$	**M1**	2.2a	7th Decompose algebraic fractions into partial fractions – repeated factors.
	Further states that: $A(3x^2-11x-4)+B(3x+1)+C(x^2-8x+16)\equiv 18x^2-98x+78$	**M1**	1.1b	
	Equates the various terms. Equating the coefficients of x^2: $3A+C=18$ Equating the coefficients of x: $-11A+3B-8C=-98$ Equating constant terms: $-4A+B+16C=78$	**M1**	2.2a	
	Makes an attempt to manipulate the expressions in order to find A, B and C. Obtaining two different equations in the same two variables would constitute an attempt.	**M1**	1.1b	
	Finds the correct value of any one variable: either $A=4$, $B=-2$ or $C=6$	**A1**	1.1b	
	Finds the correct value of all three variables: $A=4$, $B=-2$, $C=6$	**A1**	1.1b	
				(6 marks)

Notes

Alternative method

Uses the substitution method, having first obtained this equation:

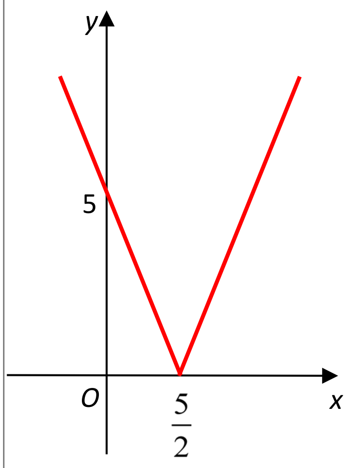
$$A(x-4)(3x+1)+B(3x+1)+C(x-4)(x-4)\equiv 18x^2-98x+78$$

Substitutes $x=4$ to obtain $13B=-26$

Substitutes $x=-\frac{1}{3}$ to obtain $\frac{169}{9}C=\frac{338}{3}\Rightarrow C=\frac{1014}{169}=6$

Equates the coefficients of x^2: $3A+C=18$

Substitutes the found value of C to obtain $3A=12$

Question	Scheme		Marks	AOs
2(a)		Correct graph in quadrant 1 and quadrant 2 with V on the *x*-axis	B1	1.1b
		States $(0, 5)$ and $\left(\frac{5}{2}, 0\right)$ **or** $\frac{5}{2}$ marked in the correct position on the *x*-axis **and** 5 marked in the correct position on the *y*-axis	B1	1.1b
			(2)	
(b)	$\lvert 2x - 5 \rvert > 7$			
	$2x - 5 = 7 \Rightarrow x = \ldots$ **and** $-(2x - 5) = 7 \Rightarrow x = \ldots$		M1	1.1b
	$\{\text{critical values are } x = 6, -1 \Rightarrow\}\quad x < -1 \text{ or } x > 6$		A1	1.1b
			(2)	
			(4 marks)	

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3a	Finds $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 12x - 12$	**M1**	1.1b	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 6x + 12$	**M1**	1.1b	
	States that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 6x + 12 \leq 0$ for all $-5 \leqslant x \leqslant -3$ and concludes this implies C is concave over the given interval.	**B1**	3.2a	
		(3)		
3b	States or implies that a point of inflection occurs when $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0$	**M1**	3.1a	7th Use second derivatives to solve problems of concavity, convexity and points of inflection.
	Finds $x = -2$	**A1**	1.1b	
	Substitutes $x = -2$ into $y = x^3 + 6x^2 - 12x + 6$, obtaining $y = 46$	**A1**	1.1b	
		(3)		
				(6 marks)

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4	Differentiates 4^x to obtain $4^x \ln 4$	**M1**	1.1b	7th Differentiate simple functions defined implicitly.
	Differentiates $2xy$ to obtain $2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$	**M1**	2.2a	
	Rearranges $4^x \ln 4 = 2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$ to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4^x \ln 4 - 2y}{2x}$	**A1**	1.1b	
	Makes an attempt to substitute (2, 4)	**M1**	1.1b	
	States fully correct final answer: $4\ln 4 - 2$ Accept $\ln 256 - 2$	**A1**	1.1b	
				(5 marks)

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5a	Writes tan*x* and sec*x* in terms of sin*x* and cos*x*. For example, $\frac{\tan x - \sec x}{1-\sin x} = \frac{\left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right)}{\left(\frac{1-\sin x}{1}\right)}$	**M1**	2.1	5th Understand the functions sec, cosec and cot.
	Manipulates the expression to find $\left(\frac{\sin x - 1}{\cos x}\right) \times \left(\frac{1}{1-\sin x}\right)$	**M1**	1.1b	
	Simplifies to find $-\frac{1}{\cos x} = -\sec x$	**A1**	1.1b	
		(3)		
5b	States that $-\sec x = \sqrt{2}$ or $\sec x = -\sqrt{2}$	**B1**	2.2a	6th Use the functions sec, cosec and cot to solve simple trigonometric problems.
	Writes that $\cos x = -\frac{1}{\sqrt{2}}$ or $x = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$	**M1**	1.1b	
	Finds $x = \frac{3\pi}{4}, \frac{5\pi}{4}$	**A1**	1.1b	
		(3)		
				(6 marks)

Question	Scheme	Marks	AOs
6	$\frac{dV}{dt} = 160\pi\,,\ V = \frac{1}{3}\pi h^2(75 - h) = 25\pi h^2 - \frac{1}{3}\pi h^3$		
	$\frac{dV}{dh} = 50\pi h - \pi h^2$	M1	1.1b
		A1	1.1b
	$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ \left(50\pi h - \pi h^2\right)\frac{dh}{dt} = 160\pi$	M1	3.1a
	When $h = 10,\ \left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\} \frac{160\pi}{50\pi(10) - \pi(10)^2}\ \left\{= \frac{160\pi}{400\pi}\right\}$	dM1	3.4
	$\frac{dh}{dt} = 0.4\ (\text{cm}\,\text{s}^{-1})$	A1	1.1b
		(5 marks)	

Question 8 Notes:

M1:	Differentiates *V* with respect to *h* to give $\pm\alpha h \pm \beta h^2,\ \alpha \neq 0,\ \beta \neq 0$
A1:	$50\pi h - \pi h^2$
M1:	Attempts to solve the problem by applying a complete method of $\left(\text{their } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 160\pi$
M1:	Depends on the previous M mark.
	Substitutes $h = 10$ into their model for $\frac{dh}{dt}$ which is in the form $\frac{160\pi}{\left(\text{their } \frac{dV}{dh}\right)}$
A1:	Obtains the correct answer 0.4

Question	Scheme	Marks	AOs
7 (a)	$y = \dfrac{3x-5}{x+1} \Rightarrow y(x+1) = 3x-5 \Rightarrow xy + y = 3x - 5 \Rightarrow y + 5 = 3x - xy$	M1	1.1b
	$\Rightarrow y + 5 = x(3-y) \Rightarrow \dfrac{y+5}{3-y} = x$	M1	2.1
	Hence $f^{-1}(x) = \dfrac{x+5}{3-x}, \quad x \in \mathbb{R},\ x \neq 3$	A1	2.5
		(3)	
(b)	$ff(x) = \dfrac{3\left(\dfrac{3x-5}{x+1}\right) - 5}{\left(\dfrac{3x-5}{x+1}\right) + 1}$	M1	1.1a
	$= \dfrac{\dfrac{3(3x-5) - 5(x+1)}{x+1}}{\dfrac{(3x-5)+(x+1)}{x+1}}$	M1	1.1b
		A1	1.1b
	$= \dfrac{9x - 15 - 5x - 5}{3x - 5 + x + 1} = \dfrac{4x-20}{4x-4} = \dfrac{x-5}{x-1}$ (note that $a = -5$)	A1	2.1
		(4)	
(c)	$fg(2) = f(4-6) = f(-2) = \dfrac{3(-2)-5}{-2+1}$;$= 11$	M1	1.1b
		A1	1.1b
		(2)	
(d)	$g(x) = x^2 - 3x = (x-1.5)^2 - 2.25$. Hence $g_{min} = -2.25$	M1	2.1
	Either $g_{min} = -2.25$ or $g(x) \geqslant -2.25$ or $g(5) = 25 - 15 = 10$	B1	1.1b
	$-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$	A1	1.1b
		(3)	
(e)	E.g. • the function g is many-one • the function g is not one-one • the inverse is one-many • $g(0) = g(3) = 0$	B1	2.4
		(1)	
			(13 marks)

Question 7 Notes:	
(a)	
M1:	Attempts to find the inverse by cross-multiplying and an attempt to collect all the *x*-terms (or swapped *y*-terms) onto one side
M1:	A fully correct method to find the inverse
A1:	A correct $\mathrm{f}^{-1}(x)=\dfrac{x+5}{3-x},\ x\in\mathbb{R},\ x\neq 3$, expressed fully in function notation (including the domain)
(b)	
M1:	Attempts to substitute $\mathrm{f}(x)=\dfrac{3x-5}{x+1}$ into $\dfrac{3\mathrm{f}(x)-5}{\mathrm{f}(x)+1}$
M1:	Applies a method of "rationalising the denominator" for both their numerator and their denominator.
A1:	$\dfrac{\dfrac{3(3x-5)-5(x+1)}{x+1}}{\dfrac{(3x-5)+(x+1)}{x+1}}$ which can be simplified or un-simplified
A1:	Shows $\mathrm{ff}(x)=\dfrac{x+a}{x-1}$ where $a=-5$ or $\mathrm{ff}(x)=\dfrac{x-5}{x-1}$, with no errors seen.
(c)	
M1:	Attempts to substitute the result of $\mathrm{g}(2)$ into f
A1:	Correctly obtains $\mathrm{fg}(2)=11$
(d)	
M1:	Full method to establish the minimum of g. E.g. • $(x\pm\alpha)^2+\beta$ leading to $g_{\min}=\beta$ • Finds the value of *x* for which $\mathrm{g}'(x)=0$ and inserts this value of *x* back into $\mathrm{g}(x)$ in order to find to $g_{\min}$
B1:	For either • finding the correct minimum value of g (Can be implied by $\mathrm{g}(x)\geqslant -2.25$ or $\mathrm{g}(x)>-2.25$) • stating $\mathrm{g}(5)=25-15=10$
A1:	States the correct range for g. E.g. $-2.25\leqslant \mathrm{g}(x)\leqslant 10$ or $-2.25\leqslant y\leqslant 10$
(e)	
B1:	See scheme