Full name:

Teacher name:

U6 A Level Maths PURE MOCK

April 2019

Time: 2 hours Total Marks: 100

You must have: **Mathematical Formulae and Statistical Tables, Calculator**

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in your name at the top of this page and the name of your teacher
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit. Answers found from the calculator without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions. The total mark for this paper is 100.
- The marks for each question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	
6	5	4	4	4	5	9	
Q8	Q9	Q10	Q11	Q12	Q13	Q14	Total
6	9	9	9	8	10	12	100
				Grade			

1. $$g(x) = \frac{2x+5}{x-3}, \quad x \geq 5.$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

(6 marks)

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2. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$.

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Given $|\overrightarrow{AC}| = 4$,

(b) find the value of a.

(3)

(5 marks)

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3. A sequence of numbers $a_1, a_2, a_3, \ldots,$ is defined by

$$a_1 = 3,$$

$$a_{n+1} = \frac{a_n - 3}{a_n - 2}, \quad n \in \mathbb{N}.$$

(a) Find $\sum_{r=1}^{100} a_r$.

(3)

(b) Hence find $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r$

(1)

(4 marks)

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4. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n (Ax + B)$$

where n, A and B are constants to be found.

(4 marks)

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5. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,dt = \ln 7,$$

show that $a = \ln k$, where k is a constant to be found.

(4 marks)

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6. $f(x) = -3x^3 + 8x^2 - 9x + 10, \; x \in \mathbb{R}.$

(a) (i) Calculate f(2).

(ii) Write f (x) as a product of two algebraic factors.

(3)

Using the answer to part (a) (ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0,$$

(2)

(5 marks)

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7. (i) Solve, for $0 \le x < \frac{\pi}{2}$, the equation

$$4 \sin x = \sec x.$$

(4)

(ii) Solve, for $0 \le \theta < 360°$, the equation

$$5 \sin \theta - 5 \cos \theta = 2,$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

(9 marks)

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8. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5.$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4].

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$.

(2)

(6 marks)

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9. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0,$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.

(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0.$$

Hence, or otherwise, find, for the adapted model,

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

(9 marks)

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10. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment, were counted.

N and T are expected to satisfy a relationship of the form

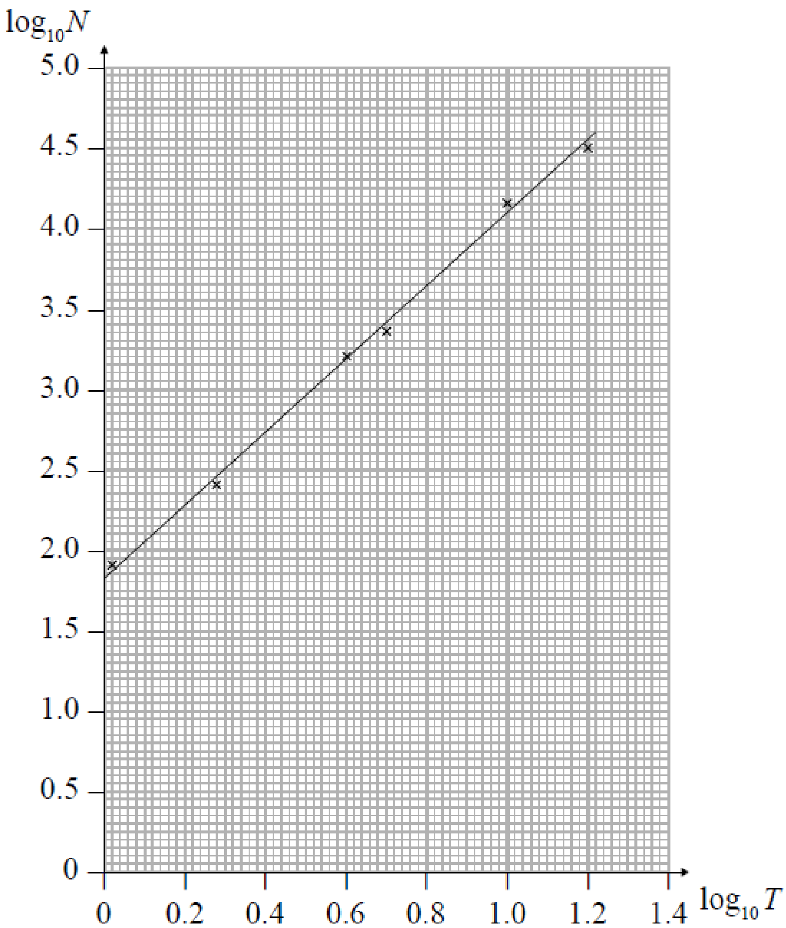
$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants.}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c,$$

giving m and c in terms of the constants a and/or b.

(2)



The diagram shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$.

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

(9 marks)

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11. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm. In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm^2, of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r}$$

(3)

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

(9 marks)

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12. Kayden claims that $3^x \geq 2^x$.

(i) Determine whether Kayden's claim is always true, sometimes true or never true, justifying your answer.

(2)

(ii) Prove that $\sqrt{3}$ is an irrational number.

(6)

(8 marks)

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13. A curve C has parametric equations

$$x = 3 + 2\sin t, \quad y = 4 + 2\cos 2t, \quad 0 \le t < 2\pi.$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$.

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2$, $x \in \mathbb{R}$.

(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

(10 marks)

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14. (a) Express $\frac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + C\,e^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found.

(3)

(12 marks)

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